9 STATICS AND TORQUE

Figure 9.1 On a short time scale, rocks like these in Australia's Kings Canyon are static, or motionless relative to the Earth. (credit: freeaussiestock.com)

Learning Objectives

9.1. The First Condition for Equilibrium
- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

9.2. The Second Condition for Equilibrium
- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

9.3. Stability
- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

9.4. Applications of Statics, Including Problem-Solving Strategies
- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

9.5. Simple Machines
- Describe different simple machines.
- Calculate the mechanical advantage.

9.6. Forces and Torques in Muscles and Joints
- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Introduction to Statics and Torque

What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton's second law states that net $F = ma$, and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

Statics

Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton's second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.

9.1 The First Condition for Equilibrium

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\text{net } \mathbf{F} = 0 \tag{9.1}$$

Note that if net F is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical *x*- and *y*-axes are zero. This is written as

$$\text{net } F_x = 0 \text{ and } F_y = 0 \tag{9.2}$$

Figure 9.2 and **Figure 9.3** illustrate situations where net $F = 0$ for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

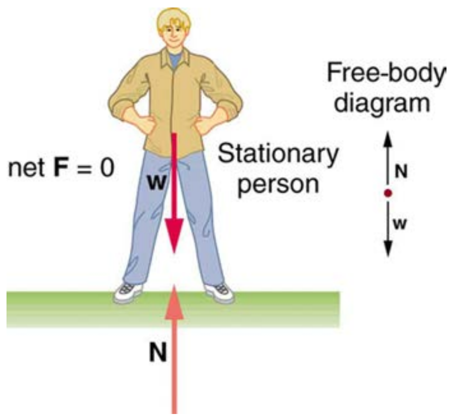

Figure 9.2 This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.

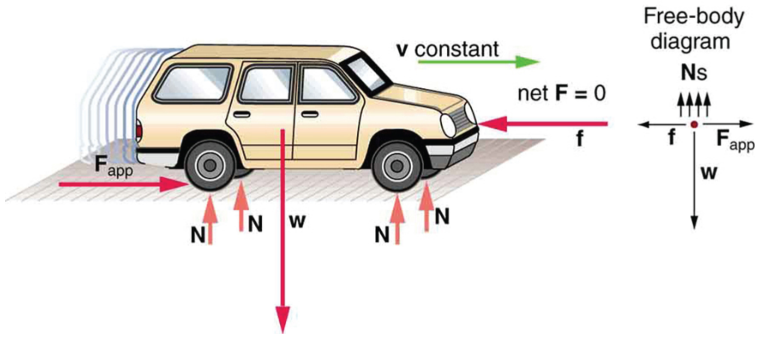

Figure 9.3 This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force F_{app} between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in **Figure 9.4** and **Figure 9.5** where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In **Figure 9.4**, the ice hockey stick remains motionless. But in **Figure 9.5**, with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

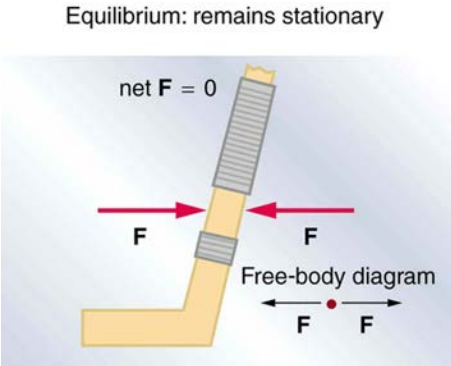

Figure 9.4 An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, $\text{net } F = 0$. Equilibrium is achieved, which is static equilibrium in this case.

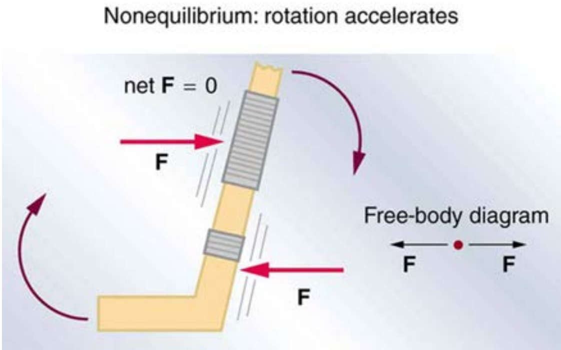

Figure 9.5 The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here $\text{net } F = 0$ but the system is *not* at equilibrium. Hence, the $\text{net } F = 0$ is a necessary—but not sufficient—condition for achieving equilibrium.

PhET Explorations: Torque

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

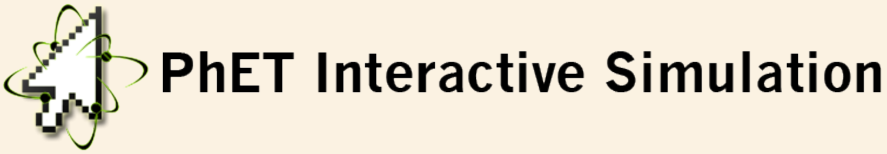

Figure 9.6 Torque (http://cnx.org/content/m42170/1.5/torque_en.jar)

9.2 The Second Condition for Equilibrium

Torque

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity. A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See **Figure 9.7**. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

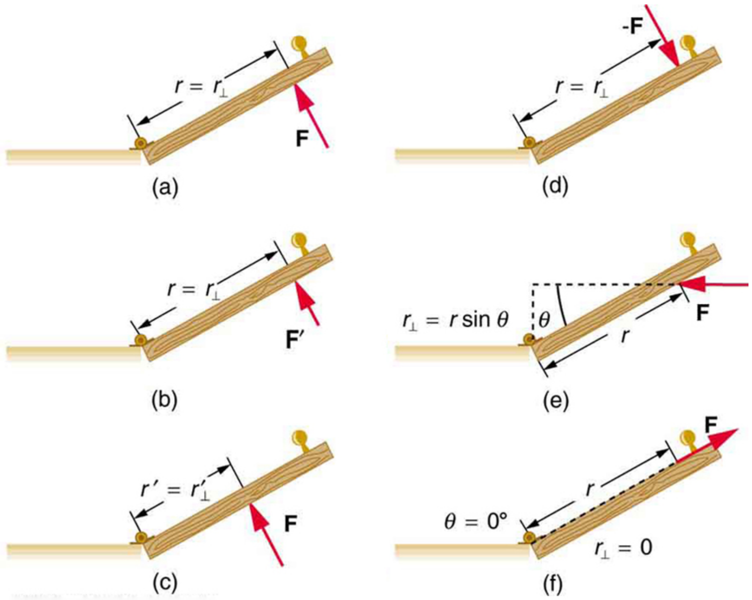

Figure 9.7 Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to $\mathbf{F}$. Note that $r_\perp$ is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force $\mathbf{F}'$ acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here, θ is less than 90^o. (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case, $\theta = 0^\mathrm{o}$.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\tau = rF \sin\theta \tag{9.3}$$

where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in **Figure 9.7** and **Figure 9.8**. An alternative expression for torque is given in terms of the **perpendicular lever arm** $r_\perp$ as shown in **Figure 9.7** and **Figure 9.8**, which is defined as

$$r_\perp = r \sin\theta \tag{9.4}$$

so that

$$\tau = r_\perp F. \tag{9.5}$$

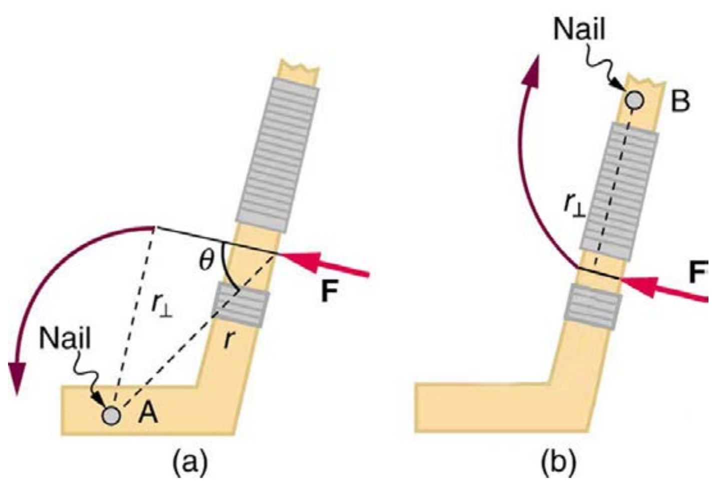

Figure 9.8 A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors r, F, and θ for pivot point A on a body are shown here— r is the distance from the chosen pivot point to the point where the force F is applied, and θ is the angle between $\mathbf{F}$ and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.

The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which $\mathbf{F}$ acts; it is shown as a dashed line in **Figure 9.7** and **Figure 9.8**. Note that the line segment that defines the distance $r_\perp$ is perpendicular to $\mathbf{F}$, as its name implies. It is sometimes easier to find or visualize $r_\perp$ than to find both r and θ. In such cases, it may be more convenient to use $\boldsymbol{\tau} = \boldsymbol{r}_\perp \boldsymbol{F}$ rather than $\tau = rF \sin\theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as $\text{N} \cdot \text{m}$. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of $32\ \text{N·m}(0.800\ \text{m}{\times}40\ \text{N}{\times}\sin 90^\text{o})$ relative to the hinges. If you reduce the force to 20 N, the torque is reduced to $16\ \text{N·m}$, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both r and θ depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen "pivot point."

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in **Figure 9.8**. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\text{net } \tau = 0 \tag{9.6}$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in **Figure 9.9**, they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.

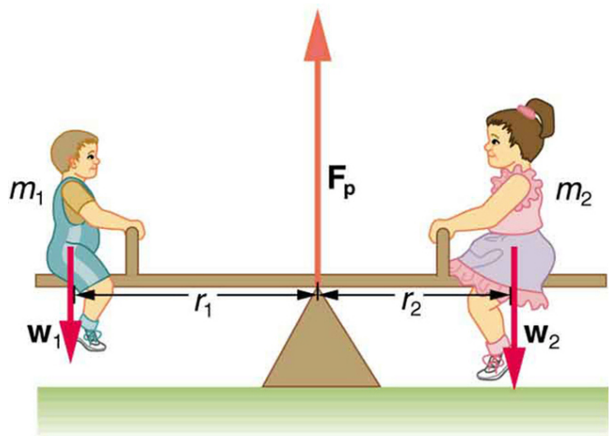

Figure 9.9 Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

Example 9.1 She Saw Torques On A Seesaw

The two children shown in **Figure 9.9** are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is F_p, the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\tau = rF \sin\theta. \tag{9.7}$$

Here $\theta = 90^\text{o}$, so that $\sin\theta = 1$ for all three forces. That means $r_\perp = r$ for all three. The torques exerted by the three forces are first,

$$\tau_1 = r_1 w_1 \tag{9.8}$$

second,

$$\tau_2 = -r_2 w_2 \tag{9.9}$$

and third,

$$\begin{aligned}\tau_\text{p} &= r_\text{p} F_\text{p} \\ &= 0 \cdot F_\text{p} \\ &= 0.\end{aligned} \tag{9.10}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since F_p acts directly on the pivot point, the distance r_p is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\tau_2 = -\tau_1, \tag{9.11}$$

or

$$r_2 w_2 = r_1 w_1. \tag{9.12}$$

Weight is mass times the acceleration due to gravity. Entering mg for w, we get

$$r_2 m_2 g = r_1 m_1 g. \tag{9.13}$$

Solve this for the unknown r_2:

$$r_2 = r_1 \frac{m_1}{m_2}. \tag{9.14}$$

The quantities on the right side of the equation are known; thus, r_2 is

$$r_2 = (1.60\text{ m})\frac{26.0\text{ kg}}{32.0\text{ kg}} = 1.30\text{ m}. \tag{9.15}$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_p. The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net }\mathbf{F} = 0. \tag{9.16}$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net }F_y = 0 \tag{9.17}$$

where we again call the vertical axis the *y*-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_\text{p} - w_1 - w_2 = 0. \tag{9.18}$$

This equation yields what might have been guessed at the beginning:

$$F_\text{p} = w_1 + w_2. \tag{9.19}$$

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_\text{p} = m_1 g + m_2 g. \tag{9.20}$$

Entering known values gives

$$\begin{aligned}F_\text{p} &= (26.0\text{ kg})\left(9.80\text{ m/s}^2\right) + (32.0\text{ kg})\left(9.80\text{ m/s}^2\right) \\ &= 568\text{ N}.\end{aligned} \tag{9.21}$$

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since F_p is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force F_p is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case*. Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation—the distances r_1 and r_2 are the distances to points directly below the **center of gravity** of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

Take-Home Experiment

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

9.3 Stability

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in **Figure 9.10**, for example, is not in stable equilibrium. There are *three types of equilibrium*: *stable*, *unstable*, and *neutral*. Figures throughout this module illustrate various examples.

Figure 9.10 presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

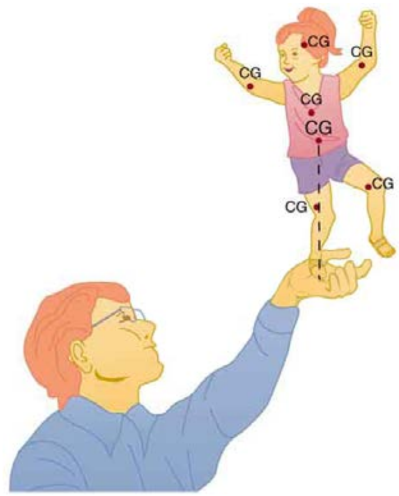

Figure 9.10 A man balances a toy doll on one hand.

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in **Figure 9.11**.

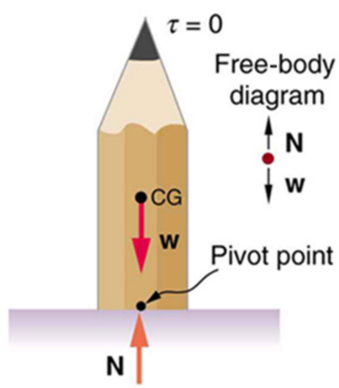

Figure 9.11 This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

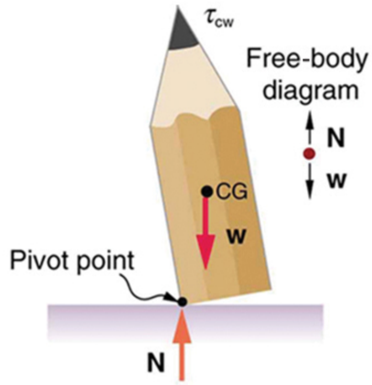

Figure 9.12 If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.

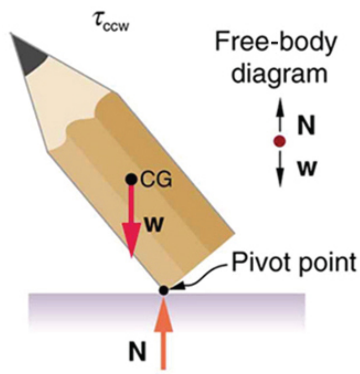

Figure 9.13 If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase.

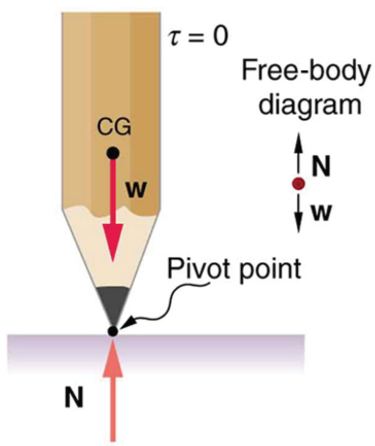

Figure 9.14 This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied.

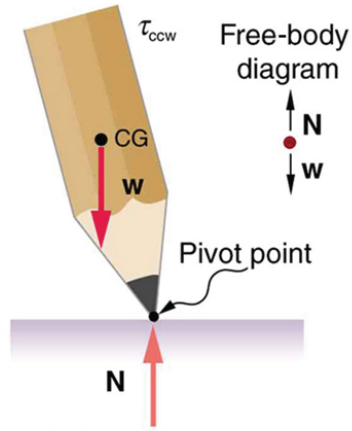

Figure 9.15 If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.

A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. **Figure 9.16** shows another example of neutral equilibrium.

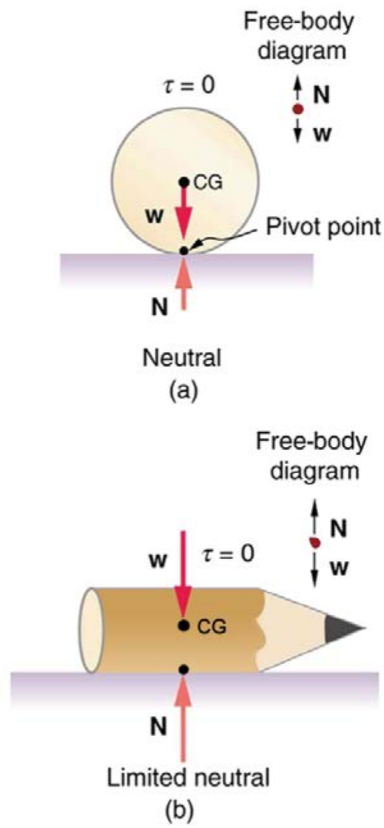

Figure 9.16 (a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil is in neutral equilibrium for displacements perpendicular to its length.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The pencil in **Figure 9.11** and the person in **Figure 9.17**(a) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

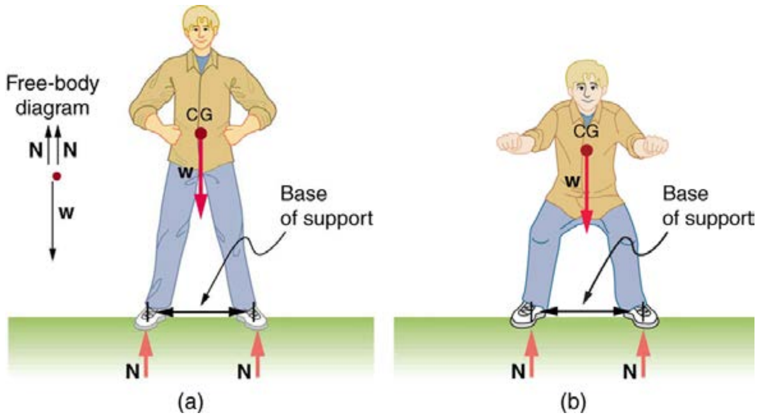

Figure 9.17 (a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.

Animals such as chickens have easier systems to control. **Figure 9.18** shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

Figure 9.18 shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

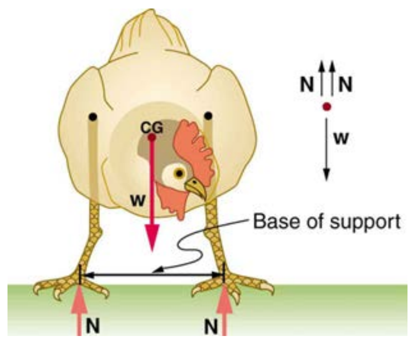

Figure 9.18 The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

Take-Home Experiment

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

9.4 Applications of Statics, Including Problem-Solving Strategies

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in **Problem-Solving Strategies**, still apply.

Problem-Solving Strategy: Static Equilibrium Situations

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.
2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations $\text{net } F = 0$ and $\text{net } \tau = 0$, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$)). Always choose a convenient coordinate system for projecting forces.

4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In **Figure 9.19**, the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium $(\text{net } F = 0)$. The second condition $(\text{net } \tau = 0)$ is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In **Figure 9.19**, a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See **Figure 9.19**. If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

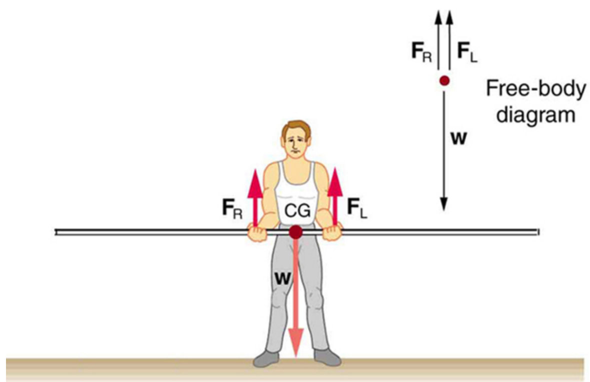

Figure 9.19 A pole vaulter holds a pole horizontally with both hands.

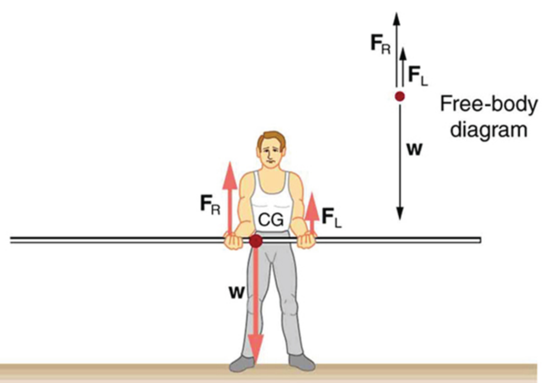

Figure 9.20 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.

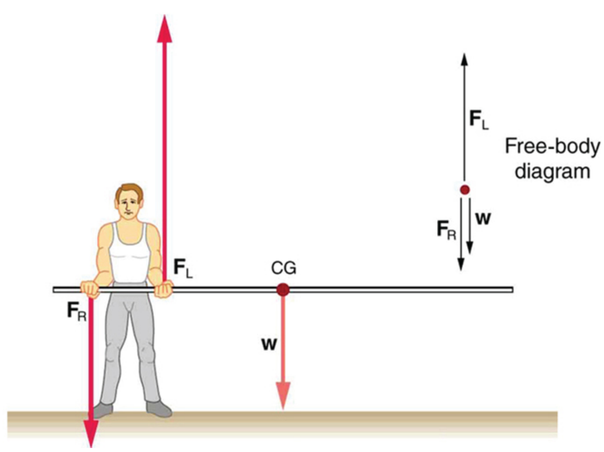

Figure 9.21 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in **Figure 9.19**, the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $F_L = F_R$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces F_L and F_R is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole (**Figure 9.21**), the direction of the force applied by the right hand of the vaulter reverses its direction.

Example 9.2 What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in **Figure 9.19**, calculate: (a) F_R , the force exerted by the right hand, and (b) F_L, the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

Figure 9.19 includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium $(\text{net } F = 0)$, since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium $(\text{net } \tau = 0)$ if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force (τ_w) and from the push or pull of the right hand (τ_R). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\text{net } \tau_\text{cw} = –\text{net } \tau_\text{ccw}. \tag{9.22}$$

or the algebraic sum of the torques is zero.

Here this is

$$\tau_R = –\tau_\text{w} \tag{9.23}$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise toque. Using the definition of torque, $\tau = rF \sin \theta$, noting that $\theta = 90º$, and substituting known values, we obtain

$$(0.900 \text{ m})(F_R) = (0.600 \text{ m})(mg). \tag{9.24}$$

Thus,

$$\begin{aligned} F_R &= (0.667)(5.00 \text{ kg})\left(9.80 \text{ m/s}^2\right) \\ &= 32.7 \text{ N}. \end{aligned} \tag{9.25}$$

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton’s second law:

$$F_L + F_R - mg = 0 \tag{9.26}$$

From this we can conclude:

$$F_L + F_R = w = mg \tag{9.27}$$

Solving for F_L, we obtain

$$\begin{aligned} F_L &= mg - F_R \\ &= mg - 32.7 \text{ N} \\ &= (5.00 \text{ kg})\left(9.80 \text{ m/s}^2\right) - 32.7 \text{ N} \\ &= 16.3 \text{ N} \end{aligned} \tag{9.28}$$

Discussion

$\boldsymbol{F_L}$ is seen to be exactly half of F_R, as we might have guessed, since F_L is applied twice as far from the cg as F_R.

If the pole vaulter holds the pole as he might at the start of a run, shown in **Figure 9.21**, the forces change again. Both are considerably greater, and one force reverses direction.

Take-Home Experiment

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

PhET Explorations: Balancing Act

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

PhET Interactive Simulation

Figure 9.22 Balancing Act (http://phet.colorado.edu/en/simulation/balancing-act)

9.5 Simple Machines

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for “machine” comes from the Greek word meaning “to help make things easier.” Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$\mathrm{MA} = \frac{F_\mathrm{o}}{F_\mathrm{i}} \tag{9.29}$$

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

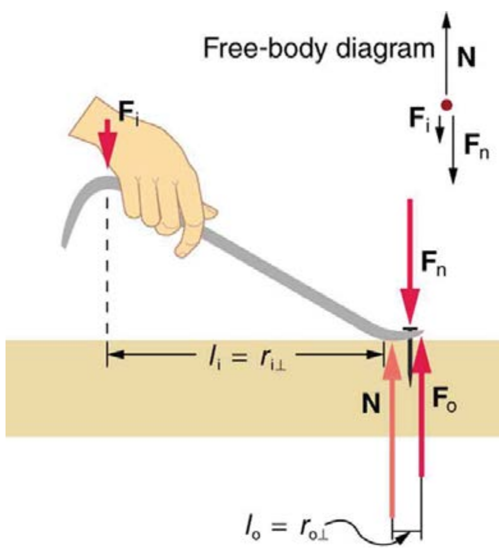

Figure 9.23 A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail ($\mathbf{F}_\mathrm{o}$) is not a force on the nail puller. The reaction force the nail exerts back on the puller ($\mathbf{F}_\mathrm{n}$) is an external force and is equal and opposite to $\mathbf{F}_\mathrm{o}$. The perpendicular lever arms of the input and output forces are l_i and l_0 .

Figure 9.23 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. $\mathbf{F}_\mathrm{i}$ is the input force and $\mathbf{F}_\mathrm{o}$ is the output force. There are three vertical forces acting on the nail puller (the system of interest) – these are $\mathbf{F}_\mathrm{i}, \mathbf{F}_\mathrm{o},$ and $\mathbf{N}$. $\mathbf{F}_\mathrm{n}$ is the reaction force back on the system, equal and opposite to $\mathbf{F}_\mathrm{o}$. (Note that $\mathbf{F}_\mathrm{o}$ is not a force on the system.) $\mathbf{N}$ is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to $\mathbf{F}_\mathrm{i}$ and $\mathbf{F}_\mathrm{n}$ must be equal to each other if the nail is not moving, to satisfy the second condition for equilibrium $(\text{net } \tau = 0)$. (In order for the nail to actually move, the torque due to $\mathbf{F}_\mathrm{i}$ must be ever-so-slightly greater than torque due to $\mathbf{F}_\mathrm{n}$.) Hence,

$$l_\mathrm{i} F_\mathrm{i} = l_\mathrm{o} F_\mathrm{o} \tag{9.30}$$

where l_i and l_o are the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_\mathrm{o}}{F_\mathrm{i}} = \frac{l_\mathrm{i}}{l_\mathrm{o}}. \tag{9.31}$$

What interests us most here is that the magnitude of the force exerted by the nail puller, F_o , is much greater than the magnitude of the input force applied to the puller at the other end, F_i . For the nail puller,

$$\text{MA} = \frac{F_\text{o}}{F_\text{i}} = \frac{l_\text{i}}{l_\text{o}}. \tag{9.32}$$

This equation is true for levers in general. For the nail puller, the MA is certainly greater than one. The longer the handle on the nail puller, the greater the force you can exert with it.

Two other types of levers that differ slightly from the nail puller are a wheelbarrow and a shovel, shown in **Figure 9.24**. All these lever types are similar in that only three forces are involved – the input force, the output force, and the force on the pivot – and thus their MAs are given by $\text{MA} = \frac{F_\text{o}}{F_\text{i}}$ and $\text{MA} = \frac{d_1}{d_2}$, with distances being measured relative to the physical pivot. The wheelbarrow and shovel differ from the nail puller because both the input and output forces are on the same side of the pivot.

In the case of the wheelbarrow, the output force or load is between the pivot (the wheel's axle) and the input or applied force. In the case of the shovel, the input force is between the pivot (at the end of the handle) and the load, but the input lever arm is shorter than the output lever arm. In this case, the MA is less than one.

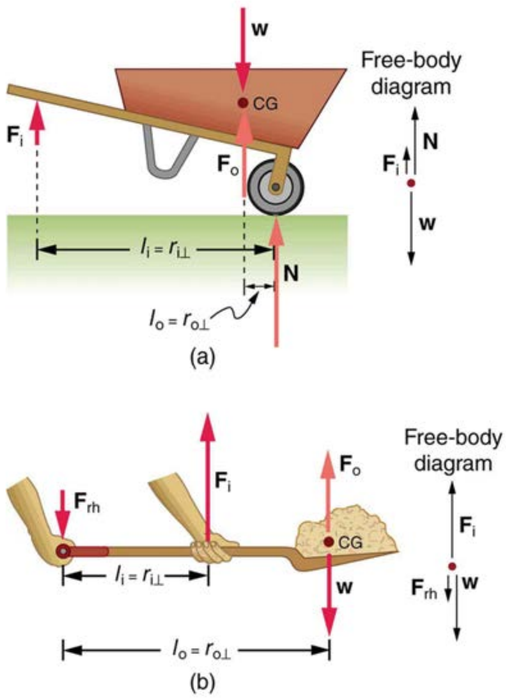

Figure 9.24 (a) In the case of the wheelbarrow, the output force or load is between the pivot and the input force. The pivot is the wheel's axle. Here, the output force is greater than the input force. Thus, a wheelbarrow enables you to lift much heavier loads than you could with your body alone. (b) In the case of the shovel, the input force is between the pivot and the load, but the input lever arm is shorter than the output lever arm. The pivot is at the handle held by the right hand. Here, the output force (supporting the shovel's load) is less than the input force (from the hand nearest the load), because the input is exerted closer to the pivot than is the output.

Example 9.3 What is the Advantage for the Wheelbarrow?

In the wheelbarrow of **Figure 9.24**, the load has a perpendicular lever arm of 7.50 cm, while the hands have a perpendicular lever arm of 1.02 m. (a) What upward force must you exert to support the wheelbarrow and its load if their combined mass is 45.0 kg? (b) What force does the wheelbarrow exert on the ground?

Strategy

Here, we use the concept of mechanical advantage.

Solution

(a) In this case, $\frac{F_\text{o}}{F_\text{i}} = \frac{l_\text{i}}{l_\text{o}}$ becomes

$$F_\text{i} = F_\text{o}\frac{l_\text{i}}{l_\text{o}}. \tag{9.33}$$

Adding values into this equation yields

$$F_\text{i} = (45.0\text{ kg})\left(9.80\text{ m/s}^2\right)\frac{0.075\text{ m}}{1.02\text{ m}} = 32.4\text{ N}. \tag{9.34}$$

The free-body diagram (see **Figure 9.24**) gives the following normal force: $F_\text{i} + N = W$. Therefore, $N = (45.0\text{ kg})\left(9.80\text{ m/s}^2\right) - 32.4\text{ N} = 409\text{ N}$. N is the normal force acting on the wheel; by Newton's third law, the force the wheel exerts on the ground is 409 N.

Discussion

An even longer handle would reduce the force needed to lift the load. The MA here is $\mathrm{MA} = 1.02 / 0.0750 = 13.6$.

Another very simple machine is the inclined plane. Pushing a cart up a plane is easier than lifting the same cart straight up to the top using a ladder, because the applied force is less. However, the work done in both cases (assuming the work done by friction is negligible) is the same. Inclined lanes or ramps were probably used during the construction of the Egyptian pyramids to move large blocks of stone to the top.

A crank is a lever that can be rotated 360^o about its pivot, as shown in **Figure 9.25**. Such a machine may not look like a lever, but the physics of its actions remain the same. The MA for a crank is simply the ratio of the radii r_i / r_0. Wheels and gears have this simple expression for their MAs too. The MA can be greater than 1, as it is for the crank, or less than 1, as it is for the simplified car axle driving the wheels, as shown. If the axle's radius is $2.0\ \mathrm{cm}$ and the wheel's radius is $24.0\ \mathrm{cm}$, then $\mathrm{MA} = 2.0 / 24.0 = 0.083$ and the axle would have to exert a force of $12{,}000\ \mathrm{N}$ on the wheel to enable it to exert a force of $1000\ \mathrm{N}$ on the ground.

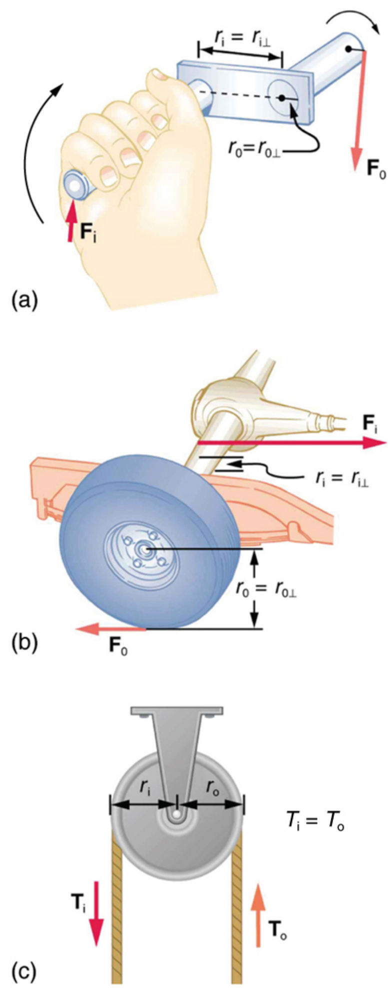

Figure 9.25 (a) A crank is a type of lever that can be rotated 360^o about its pivot. Cranks are usually designed to have a large MA. (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. The MA is less than 1. (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force T exerted by the cord without changing its magnitude. Hence, this machine has an MA of 1.

An ordinary pulley has an MA of 1; it only changes the direction of the force and not its magnitude. Combinations of pulleys, such as those illustrated in **Figure 9.26**, are used to multiply force. If the pulleys are friction-free, then the force output is approximately an integral multiple of the tension in the cable. The number of cables pulling directly upward on the system of interest, as illustrated in the figures given below, is approximately the MA of the pulley system. Since each attachment applies an external force in approximately the same direction as the others, they add, producing a total force that is nearly an integral multiple of the input force T.

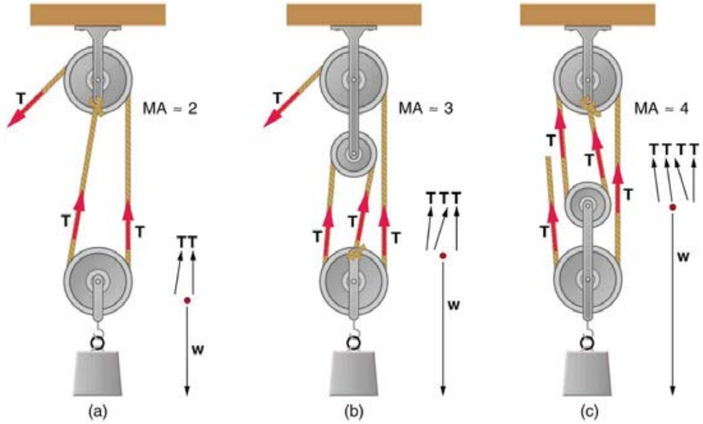

Figure 9.26 (a) The combination of pulleys is used to multiply force. The force is an integral multiple of tension if the pulleys are frictionless. This pulley system has two cables attached to its load, thus applying a force of approximately $2T$. This machine has $MA \approx 2$. (b) Three pulleys are used to lift a load in such a way that the mechanical advantage is about 3. Effectively, there are three cables attached to the load. (c) This pulley system applies a force of $4T$, so that it has $MA \approx 4$. Effectively, four cables are pulling on the system of interest.

9.6 Forces and Torques in Muscles and Joints

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. **Figure 9.27** shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in **Figure 9.27**.

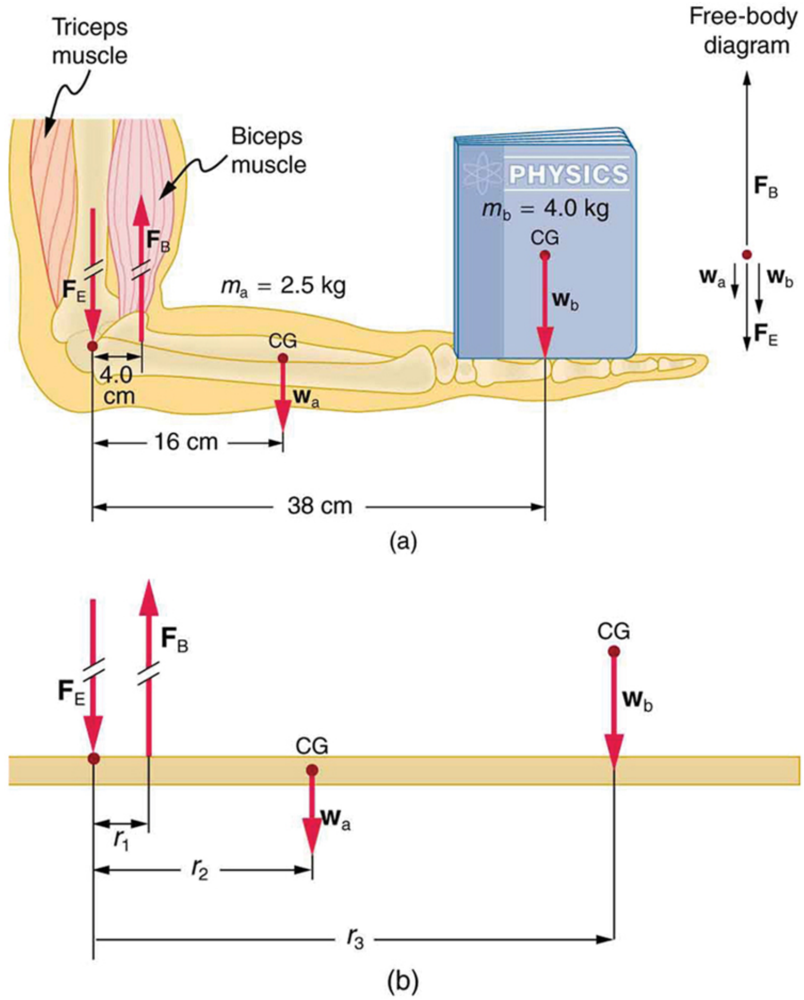

Figure 9.27 (a) The figure shows the forearm of a person holding a book. The biceps exert a force $\mathbf{F}_B$ to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in **Example 9.4**.

Example 9.4 Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in **Figure 9.27**, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is F_B; that of the elbow joint is F_E; that of the weights of the forearm is w_a, and its load is w_b. Two of these are unknown (F_B and F_E), so that the first condition for equilibrium cannot by itself yield F_B. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to F_E is zero, and the only unknown becomes F_B.

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium $(\text{net } \tau = 0)$ becomes

$$r_2 w_a + r_3 w_b = r_1 F_B. \tag{9.35}$$

Note that $\sin\theta = 1$ for all forces, since $\theta = 90^\circ$ for all forces. This equation can easily be solved for F_B in terms of known quantities, yielding

$$F_B = \frac{r_2 w_a + r_3 w_b}{r_1}. \tag{9.36}$$

Entering the known values gives

$$F_B = \frac{(0.160\text{ m})(2.50\text{ kg})\left(9.80\text{ m/s}^2\right) + (0.380\text{ m})(4.00\text{ kg})\left(9.80\text{ m/s}^2\right)}{0.0400\text{ m}} \tag{9.37}$$

which yields

$$F_\mathrm{B} = 470\ \mathrm{N}. \tag{9.38}$$

Now, the combined weight of the arm and its load is $(6.50\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) = 63.7\ \mathrm{N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_\mathrm{B}}{w_\mathrm{a} + w_\mathrm{b}} = \frac{470}{63.7} = 7.38. \tag{9.39}$$

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force F_E exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of F_E is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is, $470\ \mathrm{N} - 407\ \mathrm{N} = 63\ \mathrm{N}$, approximately equal to the weight supported.) Forces in muscles and joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as “tennis elbow,” can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the “sweet spot” on the racquet will result in little vibration or impact force being felt in the racquet and the body—less torque as explained in **Collisions of Extended Bodies in Two Dimensions**. Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

Figure 9.28 shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body’s cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body’s weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body’s cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body’s CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of **Figure 9.28**. This is compulsory; otherwise the person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in **Figure 9.29**.

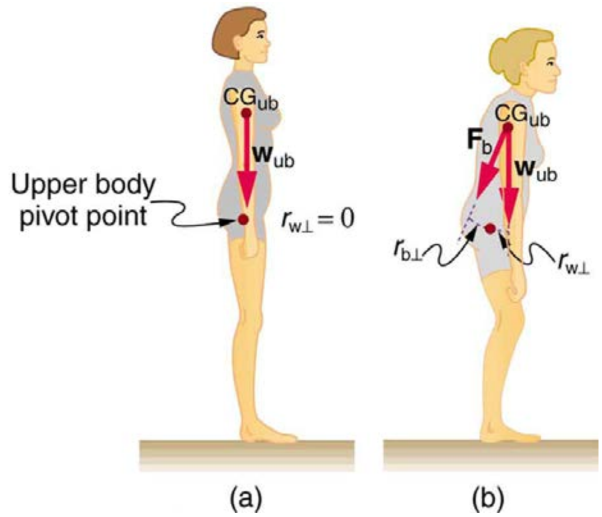

Figure 9.28 (a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm, $r_{b\perp}$, and must therefore exert a large force $\mathbf{F}_b$. Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet.

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

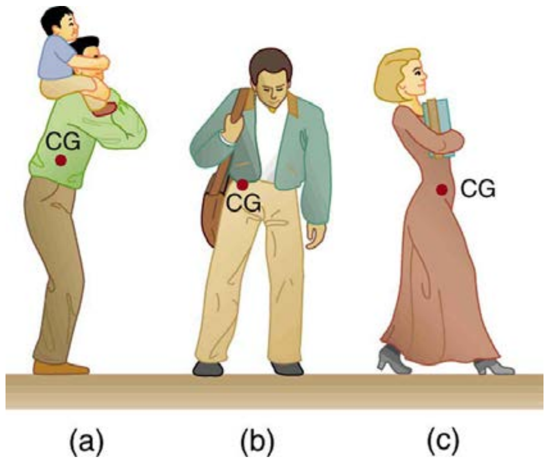

Figure 9.29 People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over his feet. (c) Another student carrying a load of books in her arms leans backward for the same reason.

Example 9.5 Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in **Figure 9.30**. (a) Calculate the magnitude of the force $F_B -$ in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude and direction of the force $\mathbf{F}_V -$ exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

Strategy

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for $F_B -$ if the pivot is chosen to be at the hips. The torques created by $\mathbf{w}_{ub}$ and $\mathbf{w}_{box} -$ are clockwise, while that created by $\mathbf{F}_B -$ is counterclockwise.

Solution for (a)

Using the perpendicular lever arms given in the figure, the second condition for equilibrium $(\text{net } \tau = 0)$ becomes

$$(0.350\text{ m})(55.0\text{ kg})\left(9.80\text{ m/s}^2\right) + (0.500\text{ m})(30.0\text{ kg})\left(9.80\text{ m/s}^2\right) = (0.0800\text{ m})F_B. \tag{9.40}$$

Solving for F_B yields

$$F_B = 4.20\times10^3\text{ N}. \tag{9.41}$$

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{F_{\mathrm{B}}}{w_{\mathrm{ub}} + w_{\mathrm{box}}} = \frac{4200\ \mathrm{N}}{833\ \mathrm{N}} = 5.04. \tag{9.42}$$

This force is considerably larger than it would be if the load were not present.

Solution for (b)

More important in terms of its damage potential is the force on the vertebrae $\mathbf{F}_{\mathrm{V}}$. The first condition for equilibrium ($\text{net } \mathbf{F} = 0$) can be used to find its magnitude and direction. Using y for vertical and x for horizontal, the condition for the net external forces along those axes to be zero

$$\text{net } F_y = 0 \text{ and net } F_x = 0. \tag{9.43}$$

Starting with the vertical (y) components, this yields

$$F_{\mathrm{V}y} - w_{\mathrm{ub}} - w_{\mathrm{box}} - F_{\mathrm{B}}\ \sin 29.0^{\mathrm{o}} = 0. \tag{9.44}$$

Thus,

$$\begin{aligned} F_{\mathrm{V}y} &= w_{\mathrm{ub}} + w_{\mathrm{box}} + F_{\mathrm{B}} \sin 29.0^{\mathrm{o}} \\ &= 833\ \mathrm{N} + (4200\ \mathrm{N}) \sin 29.0^{\mathrm{o}} \end{aligned} \tag{9.45}$$

yielding

$$F_{\mathrm{V}y} = 2.87\times10^{3}\ \mathrm{N}. \tag{9.46}$$

Similarly, for the horizontal (x) components,

$$F_{\mathrm{V}x} - F_{\mathrm{B}} \cos 29.0^{\mathrm{o}} = 0 \tag{9.47}$$

yielding

$$F_{\mathrm{V}x} = 3.67\times10^{3}\ \mathrm{N}. \tag{9.48}$$

The magnitude of $\mathbf{F}_{\mathrm{V}}$ is given by the Pythagorean theorem:

$$F_{\mathrm{V}} = \sqrt{F_{\mathrm{V}x}^{2} + F_{\mathrm{V}y}^{2}} = 4.66\times10^{3}\ \mathrm{N}. \tag{9.49}$$

The direction of $\mathbf{F}_{\mathrm{V}}$ is

$$\theta = \tan^{-1}\left(\frac{F_{\mathrm{V}y}}{F_{\mathrm{V}x}}\right) = 38.0^{\mathrm{o}}. \tag{9.50}$$

Note that the ratio of F_{V} to the weight supported is

$$\frac{F_{\mathrm{V}}}{w_{\mathrm{ub}} + w_{\mathrm{box}}} = \frac{4660\ \mathrm{N}}{833\ \mathrm{N}} = 5.59. \tag{9.51}$$

Discussion

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and load, creates much smaller forces in the back—in this case, about 5.6 times smaller.

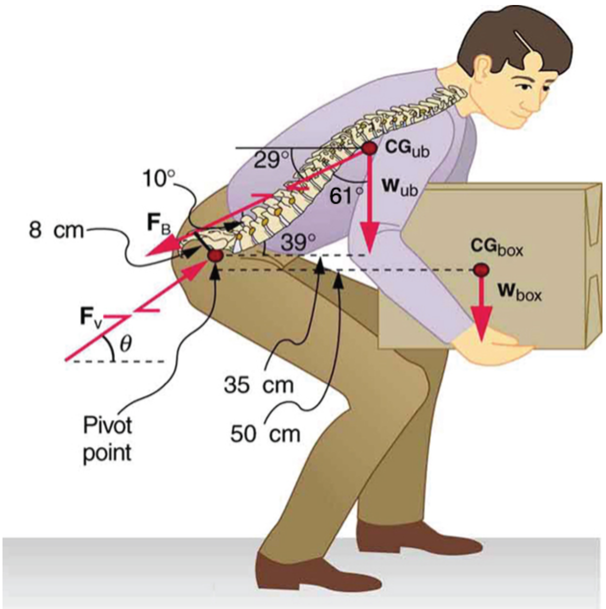

Figure 9.30 This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, **Example 9.5**.

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

Glossary

center of gravity: the point where the total weight of the body is assumed to be concentrated

dynamic equilibrium: a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

mechanical advantage: the ratio of output to input forces for any simple machine

neutral equilibrium: a state of equilibrium that is independent of a system's displacements from its original position

perpendicular lever arm: the shortest distance from the pivot point to the line along which $\mathbf{F}$ lies

SI units of torque: newton times meters, usually written as N·m

stable equilibrium: a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

static equilibrium: a state of equilibrium in which the net external force and torque acting on a system is zero

static equilibrium: equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

torque: turning or twisting effectiveness of a force

unstable equilibrium: a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

Section Summary

9.1 The First Condition for Equilibrium

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that $\text{net } \mathbf{F} = 0$.

9.2 The Second Condition for Equilibrium

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be

$$\tau = rF \sin \theta$$

where τ is torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between $\mathbf{F}$ and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm $r_\perp$ is defined to be

$$r_\perp = r \sin \theta$$

so that

$$\tau = r_\perp F.$$

- The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which F acts. The SI unit for torque is newton-meter (N·m). The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:

$$\text{net } \tau = 0$$

By convention, clockwise torques are positive, and counterclockwise torques are negative.

9.3 Stability

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

9.4 Applications of Statics, Including Problem-Solving Strategies

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in **Problem-Solving Strategies**, still apply.

9.5 Simple Machines

- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

9.6 Forces and Torques in Muscles and Joints

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints.
- Someone with good posture stands or sits in such as way that their center of gravity lies directly above the pivot point in their hips, thereby avoiding back strain and damage to disks.

Conceptual Questions

9.1 The First Condition for Equilibrium

1. What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

2. Under what conditions can a rotating body be in equilibrium? Give an example.

9.2 The Second Condition for Equilibrium

3. What three factors affect the torque created by a force relative to a specific pivot point?

4. A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.

5. Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

9.3 Stability

6. A round pencil lying on its side as in **Figure 9.13** is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?

7. Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

9.4 Applications of Statics, Including Problem-Solving Strategies

8. When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

9.5 Simple Machines

9. Scissors are like a double-lever system. Which of the simple machines in **Figure 9.23** and **Figure 9.24** is most analogous to scissors?

10. Suppose you pull a nail at a constant rate using a nail puller as shown in **Figure 9.23**. Is the nail puller in equilibrium? What if you pull the nail with some acceleration – is the nail puller in equilibrium then? In which case is the force applied to the nail puller larger and why?

11. Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

12. Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces (see previous Question)?

9.6 Forces and Torques in Muscles and Joints

13. Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

14. Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces?

15. Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?

16. Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.

17. If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.

18. Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?

19. Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

Problems & Exercises

9.2 The Second Condition for Equilibrium

1. (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

2. When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

3. Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

4. Use the second condition for equilibrium $(\text{net } \tau = 0)$ to calculate F_p in **Example 9.1**, employing any data given or solved for in part (a) of the example.

5. Repeat the seesaw problem in **Example 9.1** with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

9.3 Stability

6. Suppose a horse leans against a wall as in **Figure 9.31**. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal and opposite to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

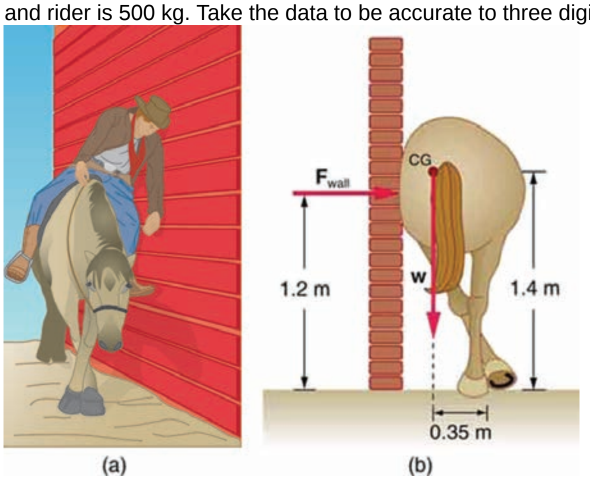

Figure 9.31

7. Two children of mass 20 kg and 30 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?

8. (a) Calculate the magnitude and direction of the force on each foot of the horse in **Figure 9.31** (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.

9. A person carries a plank of wood 2 m long with one hand pushing down on it at one end with a force F_1 and the other hand holding it up at 50 cm from the end of the plank with force F_2. If the plank has a mass of 20 kg and its center of gravity is at the middle of the plank, what are the forces F_1 and F_2?

10. A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in **Figure 9.32**. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

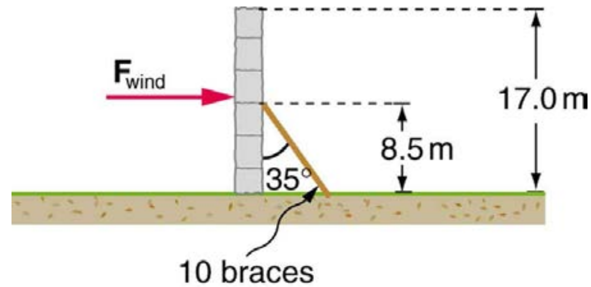

Figure 9.32

11. (a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in **Figure 9.33**? (b) What is the ratio of this force to the chicken's weight? (c) Does this support the contention that the chicken has a relatively stable construction?

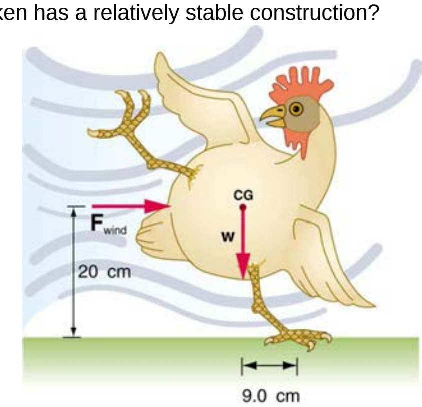

Figure 9.33

12. Suppose the weight of the drawbridge in **Figure 9.34** is supported entirely by its hinges and the opposite shore, so that its cables are slack. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances? The mass of the bridge is 2500 kg.

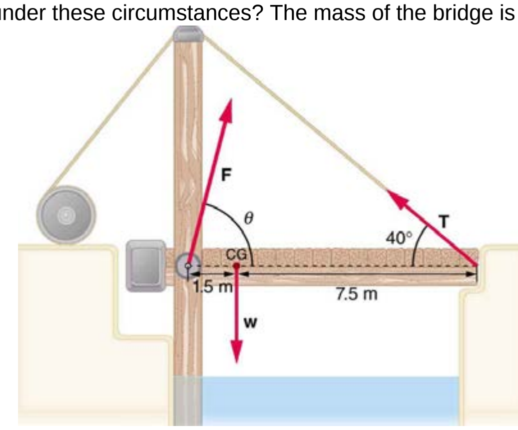

Figure 9.34 A small drawbridge, showing the forces on the hinges (F), its weight (w), and the tension in its wires (T).

13. Suppose a 900-kg car is on the bridge in **Figure 9.34** with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.

14. A sandwich board advertising sign is constructed as shown in **Figure 9.35**. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?

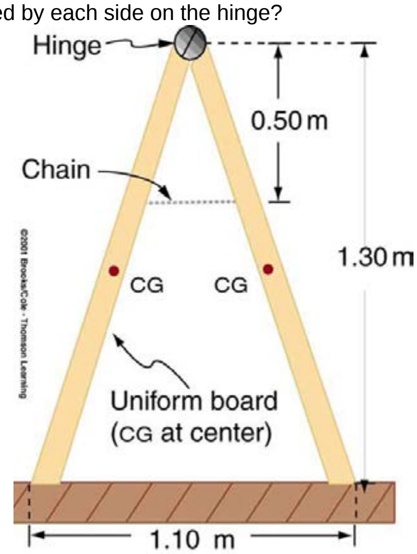

Figure 9.35 A sandwich board advertising sign demonstrates tension.

15. (a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in **Figure 9.35** in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?

16. A gymnast is attempting to perform splits. From the information given in **Figure 9.36**, calculate the magnitude and direction of the force exerted on each foot by the floor.

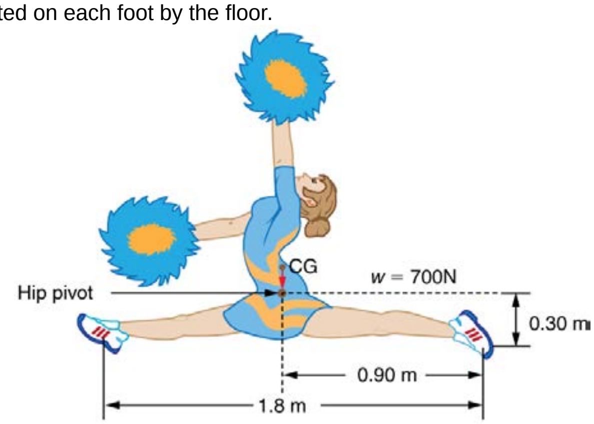

Figure 9.36 A gymnast performs full split. The center of gravity and the various distances from it are shown.

9.4 Applications of Statics, Including Problem-Solving Strategies

17. To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

18. In **Figure 9.21**, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in **Figure 9.19**, show that the second condition for equilibrium $(\text{net } \tau = 0)$ is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

9.5 Simple Machines

19. What is the mechanical advantage of a nail puller—similar to the one shown in **Figure 9.23** —where you exert a force 45 cm from the pivot and the nail is 1.8 cm on the other side? What minimum force must you exert to apply a force of 1250 N to the nail?

20. Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?

21. a) What is the mechanical advantage of a wheelbarrow, such as the one in **Figure 9.24**, if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?

22. A typical car has an axle with 1.10 cm radius driving a tire with a radius of 27.5 cm . What is its mechanical advantage assuming the very simplified model in **Figure 9.25**(b)?

23. What force does the nail puller in **Exercise 9.19** exert on the supporting surface? The nail puller has a mass of 2.10 kg.

24. If you used an ideal pulley of the type shown in **Figure 9.26**(a) to support a car engine of mass 115 kg , (a) What would be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system's mass.

25. Repeat **Exercise 9.24** for the pulley shown in **Figure 9.26**(c), assuming you pull straight up on the rope. The pulley system's mass is 7.00 kg .

9.6 Forces and Torques in Muscles and Joints

26. Verify that the force in the elbow joint in **Example 9.4** is 407 N, as stated in the text.

27. Two muscles in the back of the leg pull on the Achilles tendon as shown in **Figure 9.37**. What total force do they exert?

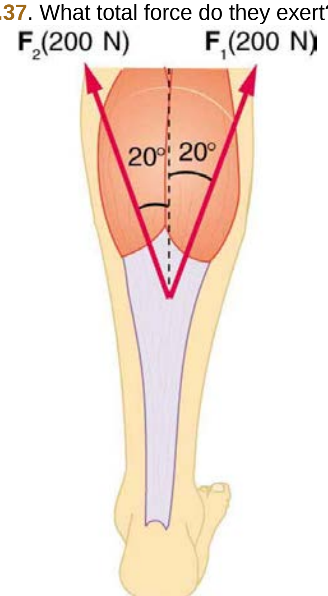

Figure 9.37 The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone.

28. The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in **Figure 9.38**. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

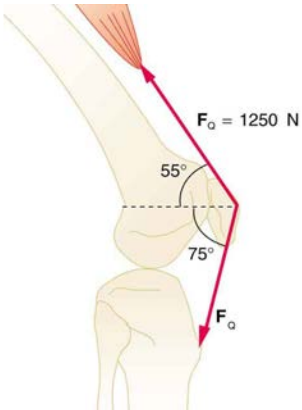

Figure 9.38 The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot.

29. A device for exercising the upper leg muscle is shown in **Figure 9.39**, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in **Applications of Statistics, Including Problem-Solving Strategies**.

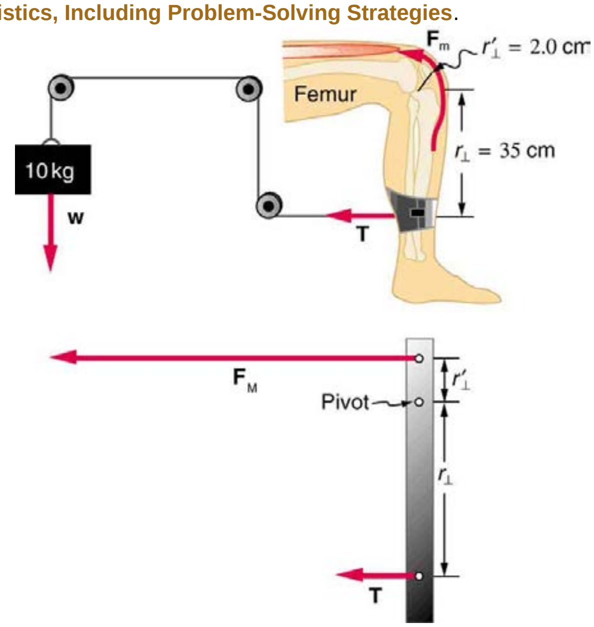

Figure 9.39 A mass is connected by pulleys and wires to the ankle in this exercise device.

30. A person working at a drafting board may hold her head as shown in **Figure 9.40**, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae $\mathbf{F}_\text{V}$ to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

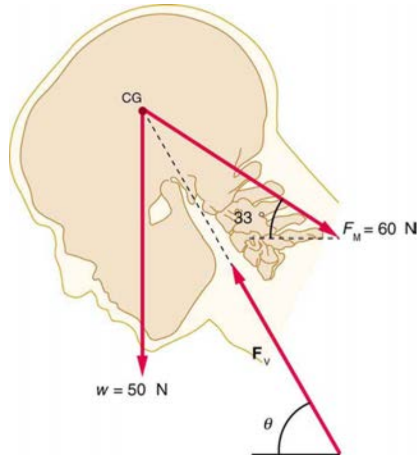

Figure 9.40

31. We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90^o. Using the same numbers as in **Example 9.4**, find the force exerted by the biceps muscle when the angle is 120^o and the forearm is in a downward position.

32. Even when the head is held erect, as in **Figure 9.41**, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

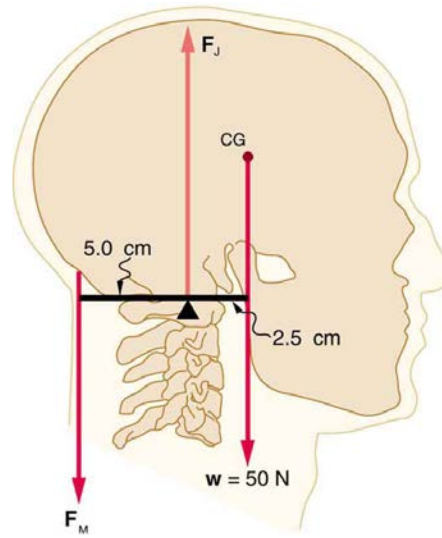

Figure 9.41 The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown.

33. A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in **Figure 9.42**. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

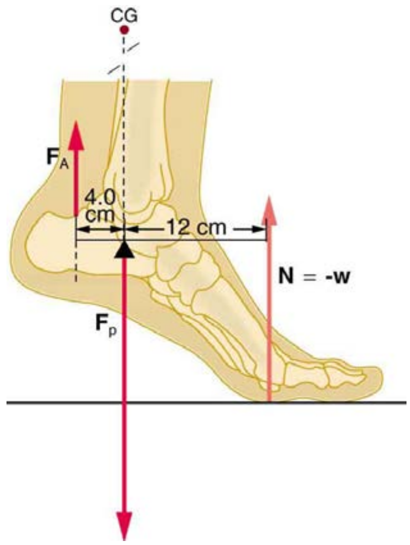

Figure 9.42 The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown.

34. A father lifts his child as shown in **Figure 9.43**. What force should the upper leg muscle exert to lift the child at a constant speed?

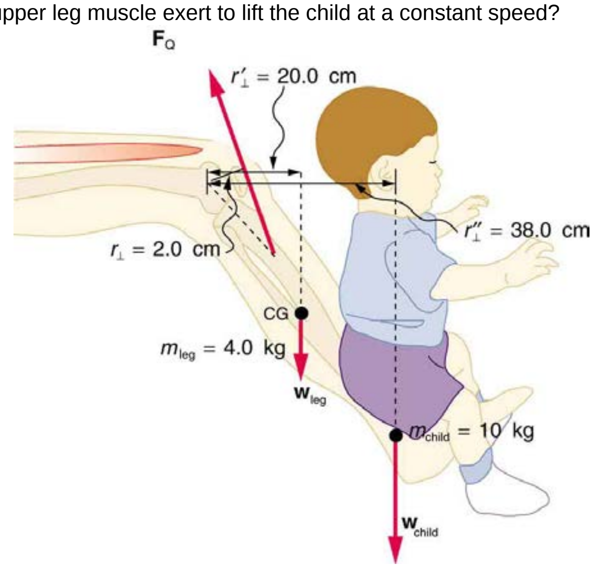

Figure 9.43 A child being lifted by a father's lower leg.

35. Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in **Figure 9.44**, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the teeth on the bullet. (b) Calculate the force on the joint.

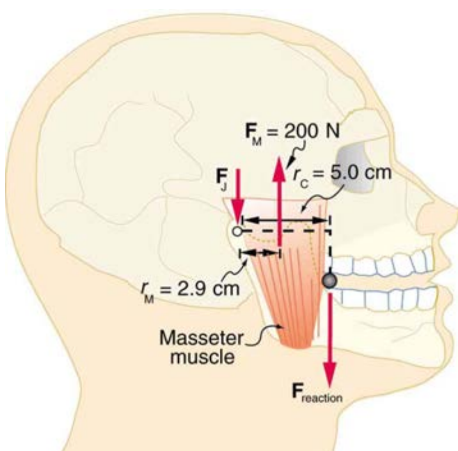

Figure 9.44 A person clenching a bullet between his teeth.

36. Integrated Concepts

Suppose we replace the 4.0-kg book in **Exercise 9.31** of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant $k = 600\text{ N/m}$. (a) How much is the rope stretched (past equilibrium) to provide the same force F_B as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25° with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

37. (a) What force should the woman in **Figure 9.45** exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

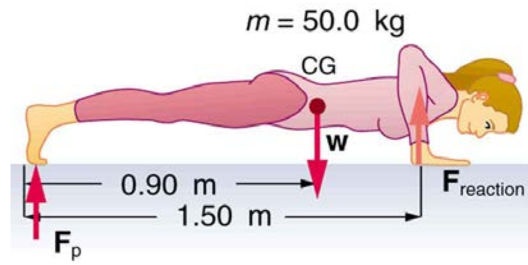

Figure 9.45 A woman doing pushups.

38. You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother's birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?

39. Unreasonable Results

Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

40. Construct Your Own Problem

Consider a method for measuring the mass of a person's arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its center of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.